Announcements

- HW8 divide and conquer, due Friday April 10
- Section survey responses

Plan for the remainder of the semester

- HW9-11 due Fridays April 17, 24, May 1
- Cumulative Final, May 9th

Bowers CIS Pre-Enroll Event

CornellBowers
College of Computing + Information Science

Open to all CS/IS/ISST pre-majors, majors, minors, and anyone interested in computing.

→ April 13, 2026
4:00 – 5:30 PM
CIS Building, Wayfair Commons (Room 132), Conine and Shah Families Active Learning Classroom (Room 142)

WICC, URMC, ACSU, ISSA, Queery

Algorithms dealing with NP-complete problems

Approximation Algorithm using linear programs

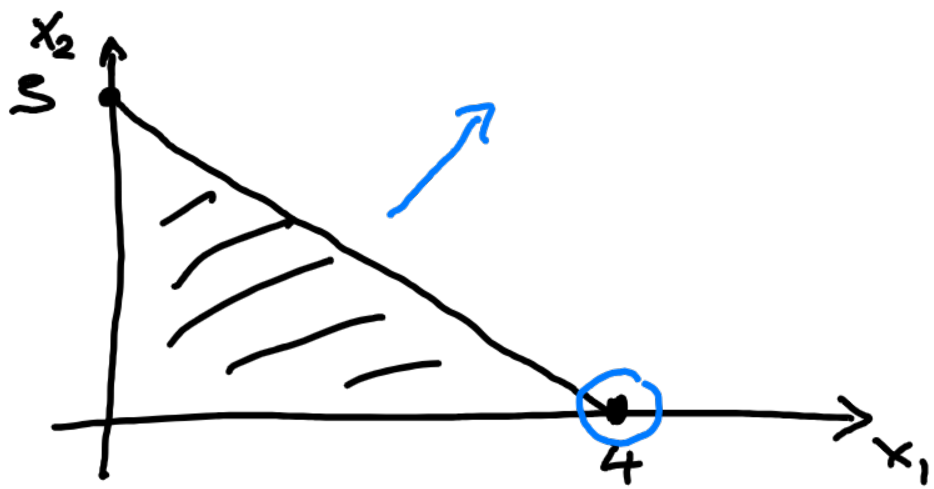
Linear program:

variables $x_1 \dots x_n$ real (fractions OK)

$Ax \le b$ constraints

- find x satisfying all this
- find x minimizing $\sum_i c_i x_i$

x_1 & x_2



$x_1 \ge 0$
$x_2 \ge 0$

$3x_1 + 4x_2 \le 12$

max $x_1 + x_2$ $\longrightarrow$ solution $(4, 0)$

Analog: linear algebra

$Ax = b$

Linear programming (cont.)

1. Solvable in polynomial time   Khachiyan 1979
2. ORIE 3300
3. great software available

   uses simplex very fast almost always
   not polynomial in the worst case

Plan for 4820: learn to use this as a tool

today: using fact 1: in obtaining approximate solutions

see more Monday

0-1 Integer programming is NP-hard

$x_1 \dots x_n \qquad x_i \in \{0,1\}$

$Ax \leq b$

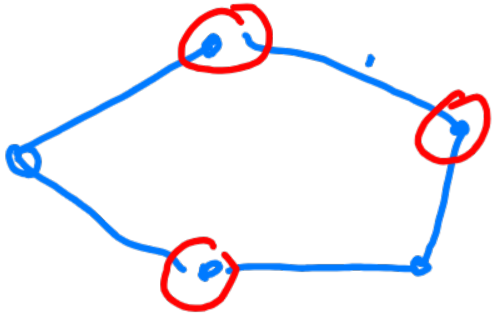
minimize $\sum c_i x_i$

Claim: Vertex Cover $\leq$ 0-1 Integer linear programming

Input: $G=(V,E)$ & k

$\exists\ S \subseteq V \quad |S|=k$

all edges $e=(v,u)$ either v or $u \in S$



Idea: x_v all $v \in V$

$x_v = 1$ if $v \in S$

$\ 0$ if $v \notin S$

$0 \leq x_v \leq 1 \quad \forall v \in V$

$x_v + x_u \geq 1 \quad e=(u,v) \in E$

$\min \sum_v x_v$

0-1 Integer programming is NP-hard (cont.)

Claim: min in 0-1 Int. program $\leq k$ if & only Vertex Cover yes

Proof if Vertex cover S exist: $x_v = \begin{cases} 1 & v \in S \\ 0 & v \notin S \end{cases}$

$\sum_v x_v = |S| = k$ & all ineq. satisfied

if solution value $\leq k$

set $S = \{v : x_v = 1\}$

(1) S is vertex cover

$x_v + x_u \geq 1$

$\Rightarrow$ S covers edge

$\Rightarrow \sum_v x_v \leq k \Rightarrow |S| \leq k$

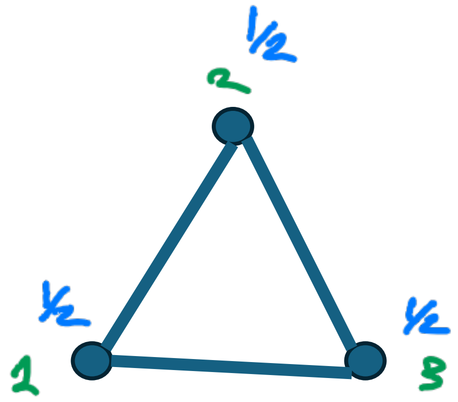
Con min cost version: min $\sum_{v \in S} c_i$ Vertex cover min cost

Join by Web **PollEv.com/evatardos772**

What is the optimal value of the vertex cover linear program on a triangle?

A. 1
B. 1.5
C. 2
D. 3
E. None of these

$x_i = \frac{1}{2}$ all i



$$1 \geq x_1, x_2, x_3 \geq 0$$
$$x_1 + x_2 \geq 1$$
$$x_2 + x_3 \geq 1$$
$$x_1 + x_3 \geq 1$$
$$\min x_1 + x_2 + x_3$$

solution with integers
need to choose 2 vertices

solve as linear program
fractional values OK

Plan of using Linear Programming for approximation

① Set up integer version equivalent to Vertex cover

$0 \leq x_v \leq 1 \quad \forall v$

$x_v + x_u \geq 1 \quad \forall e = (u, v)$

② Solve it as linear program

get answer in fractions $0 \leq x_v \leq 1$ for $v \in V$

③ round the fraction

$S = \{ v : \quad x_v \geq 1/2 \}$

Claim 1: S is vertex cover

edge $e = (v, u)$

we have

$x_v + x_u \geq 1 \implies$ either u or v

$x_u \geq 1/2$ or $x_v \geq 1/2$

$\implies$ S covers e

Approximation Algorithm

Recall optimization version of vertex cover

$\min |S|$

or $\min \sum_{v \in S} w_v$

Recall finding true optimal $\min |S|$ is NP-hard

need good bound: Opt for Vertex cover $\geq$??

Claim 2: linear programming Optimum $\leq$ min vertex cover

$$\min \sum_v x_v \quad 0 \leq x_v \leq 1 \ \forall v \quad x_u + x_v \geq 1 \ \forall e=(u,v)$$

$$\leq$$

$$\min \sum x_v \quad 0 \leq x_v \leq 1 \ \forall v \quad x_u + x_v \geq 1 \ \forall e=(u,v) \quad x_v \in \{0,1\}$$

Claim 3: Rounding alg is 2-approximation

Approximation Algorithm

S is vertex cover by (1)

$|S| = |\{v : x_v \geq 1/2\}| \leq 2 \sum_{v \in S} x_v = 2$ linear programming optimum $\geq 2 \cdot Opt$

equal $v : x_v = 1/2$

sharp $v : x_v$

claim(2)